ASD TECHNICAL REPORT 61-63

ELASTIC-PLASTIC DEFORMATION OF A SINGLE GROOVED FLAT PLATE UNDER LONGITUDINAL SHEAR

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DECEMBER 1961

AERONAUTICAL SYSTEMS DIVISION

NOTICES

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FOREWORD

This report was prepared by the Department of Mechanical Engineering, Massachusetts Institute of Technology, under Contract No. AF 18(600)-957. The contract was initiated under Project No. 7351, "Metallic Materials," Task No. 73521, "Behavior of Metals." The work was monitored by the Metals and Ceramics Laboratory, Directorate of Materials and Processes, Deputy for Technology, Aeronautical Systems Division, with Mr. D. M. Forney, Jr. acting as project engineer.

This report covers the period of work from September 1959 to September 1961.

ABSTRACT

The development of the plastic strain in a V-grooved flat plate under longitudinal shear was followed from the elastic through the partially plastic to the fully plastic condition for a non-strainhardening material. The region of plastic flow develops monotonically. Adjacent to the zone of deformation in the fully plastic case there is a region where limited plastic deformation has occurred.

The results for the growth of the plastic zone were compared with predictions based on the elastic-plastic solution for an infinite plate and the elastic solution for a finite plate. Agreement is good at low stress levels. At high stress levels, a relatively simple empirical equation, satisfying overall equilibrium, is proposed. Predictions based on elasticity theory alone are shown to be seriously in error.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

W. J. Trapp

W. J. Trapp
Chief, Strength and Dynamics Branch
Metals and Ceramics Laboratory
Directorate of Materials and Processes

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I. INTRODUCTION

For an understanding of fatigue it is desirable to know the distribution of plastic yielding at the tip of a groove or growing fatigue crack. Since problems in longitudinal shear are more readily solved than others, it was decided to study a circumferentially notched, thin-walled tube loaded in torsion, which could be idealized as a grooved plate loaded in longitudinal shear, as shown in Fig. 1. The plate, containing an infinite V-groove in its face, is loaded with a shear stress τ_∞ parallel to the groove. The plate thickness is t, the groove depth is c, and the semi-groove angle is α . The material is elastic-plastic, non-strainhardening with a yield stress in shear of k.

II. ELASTIC-PLASTIC ANALYSIS

Consideration of the case of an infinitely thick plate, given by Hult and McClintock (1956), or of the limiting case of torsion as given by Prager and Hodge (1951), leads one to postulate displacement only in the direction of the axis of the groove. The equations of equilibrium are satisfied if the components of stress are set equal to derivatives of a potential function:

$$\tau_{xz} = \frac{\partial \phi}{\partial y} \quad ; \quad \tau_{yz} = -\frac{\partial \phi}{\partial x} \quad . \tag{1}$$

In the elastic region Hooke's Law and the dependence of strains on the one component of displacement then lead to the familar Laplace equation for the stress potential ,

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad , \tag{2}$$

subject to the boundary condition,

$$\phi(x,y) = \int^{x,y}\left(\frac{\partial \phi}{\partial x}\,dx + \frac{\partial \phi}{\partial y}\,dy\right) = \int^{x,y}\left(\tau_{xz}\,dy - \tau_{yz}\,dx\right), \tag{3}$$

integrated around the boundary of the elastic region.

Equation 2 with its associated boundary condition, Eq. 3, applies only in the region where Hooke's Law is satisfied and can be solved only with a knowledge of the position of the elastic-plastic boundary. (More precisely, it is not the plasticity but the non-linearity of the stress strain relation which is critical in determining this boundary,

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but the two conditions are identical for the material considered here.) The need to know the elastic-plastic boundary is avoided by transforming from space coordinates to stress coordinates. Since the Jacobian of τ_{xz} and τ_{yz} with respect to x and y is non-zero, the continuity and equilibrium equations can be written as

$$\left(\frac{\partial x}{\partial \tau_{yz}}\right)_{\tau_{xz}} - \left(\frac{\partial y}{\partial \tau_{xy}}\right)_{\tau_{yz}} = 0$$
$$\left(\frac{\partial x}{\partial \tau_{xz}}\right)_{\tau_{yz}} + \left(\frac{\partial y}{\partial \tau_{yz}}\right)_{\tau_{xz}} = 0 \quad . \tag{4}$$

The second of these will be identically satisfied if the coordinates are set equal to the partial derivatives of a potential with respect to the components of stress:

$$x = \left(\frac{\partial \psi}{\partial \tau_{yz}}\right)_{\tau_{xz}}$$
$$y = -\left(\frac{\partial \psi}{\partial \tau_{xz}}\right)_{\tau_{yz}} \quad . \tag{5}$$

From Eq. 4, the coordinate potential ψ then must satisfy Laplace's equation,

$$\frac{\partial^2 \psi}{\partial \tau_{xz}^2} + \frac{\partial^2 \psi}{\partial \tau_{yz}^2} = 0 \quad , \tag{6}$$

subject to the boundary condition

$$\psi(\tau_{xz}, \tau_{yz}) = \int^{\tau_{xz}, \tau_{yz}} \left(\frac{\partial \psi}{\partial \tau_{xz}} d\tau_{xz} + \frac{\partial \psi}{\partial \tau_{yz}} d\tau_{yz}\right)$$
$$= \int^{\tau_{xz}, \tau_{yz}} \left(x\, d\tau_{xz} - y\, d\tau_{yz}\right) \quad . \tag{7}$$

This transformation and the resulting boundary conditions are shown in Fig. 1. In integrating around the elastic-plastic boundary it must be noted that the resultant stress is equal to the yield stress and is normal to a position vector from the tip of the groove. The entire boundary except the magnitude of the stress at the free surface opposite the groove (τ_b) is known. However, from the continuity of stress in space it can be shown that $(\partial\psi/\partial\tau_{xz})$ is continuous in the τ_{yz} - direction across the point ($\tau_{xz} = 0, \tau_{yz} = \tau_b$), which is sufficient information to uniquely determine a solution.

Due to this indeterminate boundary condition, the author was not able to find a closed form, analytic solution to the problem. Therefore the solution was carried out by machine computation[1] using relaxa-

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[1] These computations were performed in part at the M. I. T. Computation Center, Cambridge, Mass., and completed at the University Mathematical Laboratory, Cambridge, England.

tion techniques plus a provision for satisfying continuity of $(\partial\psi/\partial\tau_{YZ})$ across $\tau_{YZ} = \tau_b$. To simplify computation, the sector-shaped area of the stress plane was mapped conformally into a semi-infinite strip. Since the zero stress point of the stress plane is a singular point of this transformation, the boundary there was replaced by a small circle of radius $\tau_{\infty}\exp(-3\pi/2)$ and ψ was simply called zero on this circle. The number of cycles to obtain proper convergence and the accuracy of the finite difference approximations were determined as outlined by Crandall (1956). The degree of convergence was checked by performing extra cycles in one case. The slopes of the potential function are accurate to about 5% and are on the low side of the correct value. The results of these computations are shown in Figs. 3 through 8.

Hult and McClintock (1956) have shown that in the plastic zone, the strain consists of a single shear component normal to the position vector, $\mathbf{r}$. This shear component is given in terms of the yield strain, $\gamma_Y = k/G$, and the distance to the plastic-elastic boundary (R), measured through the point in question, by

$$\gamma_{\theta z} = \gamma_Y R/r \quad . \tag{8}$$

Therefore since the elastic-plastic boundary is known, the strain distribution in the plastic zone can be determined.

It can be seen from Fig. 6 that for any element, the strains are monotonically increasing functions of the applied load. Therefore the same result is obtained for either non-linearly elastic or elastic-plastic material.

Once the fully plastic state is reached, all further deformation takes place on the plane y = 0, as predicted by a rigid-plastic analysis. It is observed that adjacent to the plane of deformation in the fully plastic state, a region of limited deformation exists as suggested by Hill (1950).

III. COMPARISON WITH ELASTIC ANALYSIS

Since numerical computations are time consuming and do not afford a compact summary of results, it is well to inquire how closely these results might be anticipated from an elastic analysis coupled with the elastic-plastic analysis of Hult and McClintock (1956) for a plate of infinite thickness.

As shown by Williams (1957) for zero-angle notches under tension or transverse shear, the stress distribution near the tip of the notch in a body of arbitrary shape can be characterized by a single parameter. Similarly, for a zero-angle groove, or crack, under longitudinal shear, Hult and McClintock ([illegible]) found the elastic stress distribution to be

$$\tau_{\theta z} = K_3 r^{-1/2} \cos\frac{\theta}{2} \; ; \; \tau_{rz} = K_3 r^{-1/2} \sin\frac{\theta}{2} \quad . \tag{9}$$

For the infinite plate, the constant K_3 is given in terms of the stress at infinity (τ_∞) and the crack depth (c) by

$$K_{3\infty} = \tau_\infty (c/2)^{1/2}. \tag{10}$$

For a material with yield strength in shear of k, the radius of the plastic zone can be found from the case of a crack in an infinite solid to be given in terms of the elastic stress intensity factor, K_3, of the elastic solution by

$$R = 2K_3^2/k^2 = c\,(\tau_\infty/k)^2 \, . \tag{11}$$

The elastic stress intensity factor for the plate of finite thickness, K_{3t}, is found by a transformation similar to that used by Westergaard ([illegible]) for the case of a series of cracks in a plate under tension. The transformation yields a correction for plate thickness identical to that found by Westergaard, which is also recommended by the ASTM (1960) for tension of sheets of finite width:

$$\left(K_{3t}/K_{3\infty}\right)^2 = \frac{2\pi}{\pi c} \tan\frac{\pi c}{2t} \quad . \tag{12}$$

In the case of shear, this elastic correction factor is exact. In the case of a plate under tension it is not, because normal stress exists on the lines lying between the cracks in Westergaard's analysis, but not on the lateral boundaries of the actual separate sheets.

In the case of plastic flow, as the applied stress is increased the plastic zone grows more rapidly than given by Eqs. 11 and 12. An approximate equation can be formulated by noting that when the minimum section becomes fully plastic, equilibrium requires that the applied stress be $\tau_\infty = k(t-c)/t$. This can be achieved by adding a correction term to Eqs. 11 and 12:

$$R_0 = c\left(\frac{\tau_\infty}{k}\right)^2\left(\frac{2t}{\pi c}\tan\frac{\pi c}{2t}\right) + C'\left(1-\sqrt{1-\left(\frac{\tau_\infty}{k}\frac{t}{t-c}\right)^N}\right) \quad . \tag{13}$$

Choosing the exponent on the stress term to be 5, and the constant C to give the proper limit gives

$$C' = (t-c)\left(1 - \frac{2(t-c)}{\pi t}\tan\frac{\pi c}{2t}\right) \quad . \tag{14}$$

The results of the approximate relation given by Eq. 13 are plotted in Figs. 3 through 5 and give satisfactory agreement with the more exact numerical solution. It is also of interest to compare these results with the assumption that the plastic zone extends to the point where the stress calculated on an elastic basis reaches the yield stress, as recommended by the ASTM (1960). In making the correction for the finite width of a plate, they suggest that the effective crack length should be taken to be the actual crack length plus the radius from the tip of the crack to the elastic-plastic boundary directly ahead of the crack, $c + R_0$. These ideas lead to the equation

$$R_0 = \frac{c}{2}\left(\frac{\tau_\infty}{k}\right)^2\frac{2t}{\pi c}\tan\frac{\pi(c+R_0)}{2t} \quad . \tag{15}$$

As shown in Figs. 3 through 5, this approximation is unsatisfactory. In the first place, at low stress levels it was shown by Hult and McClintock (1956) that this type of approximation gave results low by a factor of two. On the otherhand, at high stress levels the equation is unsatisfactory because it implies that when the plastic zone reaches all the way across the plate, the load carrying capacity has dropped to zero. Even without the appearance of the term for the radius of the plastic zone in the correction for finite plate width, the equation fails to satisfy equilibrium in the fully plastic case. Since this objection holds in the case of a sheet under tension as well as a plate under shear, the tensile analog of Eq. 13 is to be preferred.

A further disadvantage to the above assumption, that the boundary of the plastic zone is the point at which the elastic stress distribution reaches the yield stress, is seen on noting that such an assumption would indicate plastic flow occurring along the flank of the groove, whereas as shown in Figs. 6 through 8, the plastic zone actually lies entirely ahead of the crack.

IV. CONCLUSIONS

1) In the fully plastic case, the strain in a singly-grooved flat plate under longitudinal shear is concentrated along the minimum section. As the plastic flow develops, however, small scale plastic strain occurs in a monotonically increasing region on either side of the plane of minimum cross section.

2) The extent of the plastic zone ahead of the groove, R_0, is given in terms of the plate thickness t, the crack length or groove depth x, the nominal applied stress, τ_∞ , and the yield stress in shear, k, by the equation

$$R_0 = c\left(\frac{\tau_\infty}{k}\right)^2 \frac{2t}{\pi c} \tan\frac{\pi c}{2t} + c'\left(1 - \sqrt{1 - \left(\frac{\tau_\infty}{k}\,\frac{t}{t-c}\right)^5}\,\right) ,$$

where the constant c', chosen to satisfy equilibrium in the fully plastic case, is

$$c' = (t-c)\left(1 - \frac{2(t-c)}{\pi t} \tan\frac{\pi c}{2t}\right) .$$

3) A corresponding equation, derived by a method recommended by the ASTM, is in error by a factor of two at low stress levels, and is even qualitatively incorrect at high stress levels.

4) Within the plastic zone, the strain is given in terms of the radius to the elastic-plastic boundary by

$$\gamma_{\theta z} = \frac{k}{G}\left(\frac{r}{R}\right) .$$

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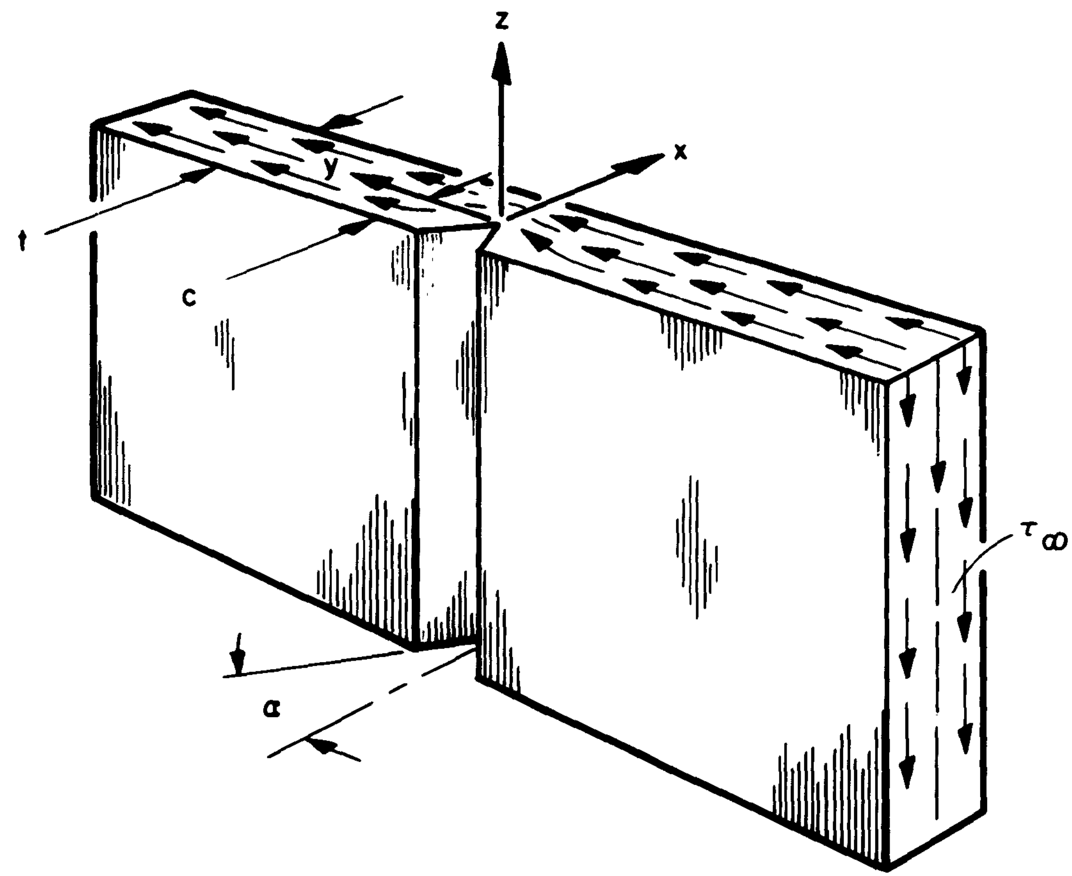

FIG. 1 GROOVED PLATE LOADED IN LONGITUDINAL SHEAR.

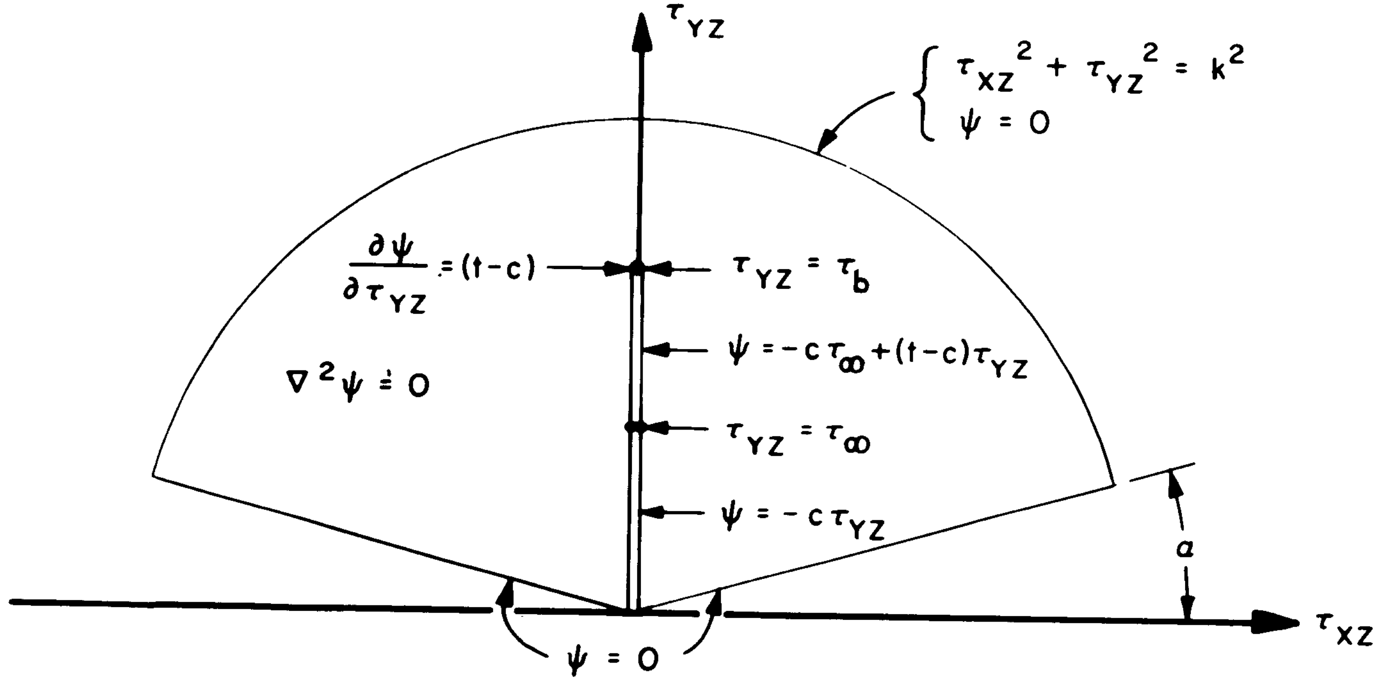

FIG. 2 ELASTIC-PLASTIC PROBLEM IN STRESS COORDINATES WITH BOUNDARY CONDITIONS.

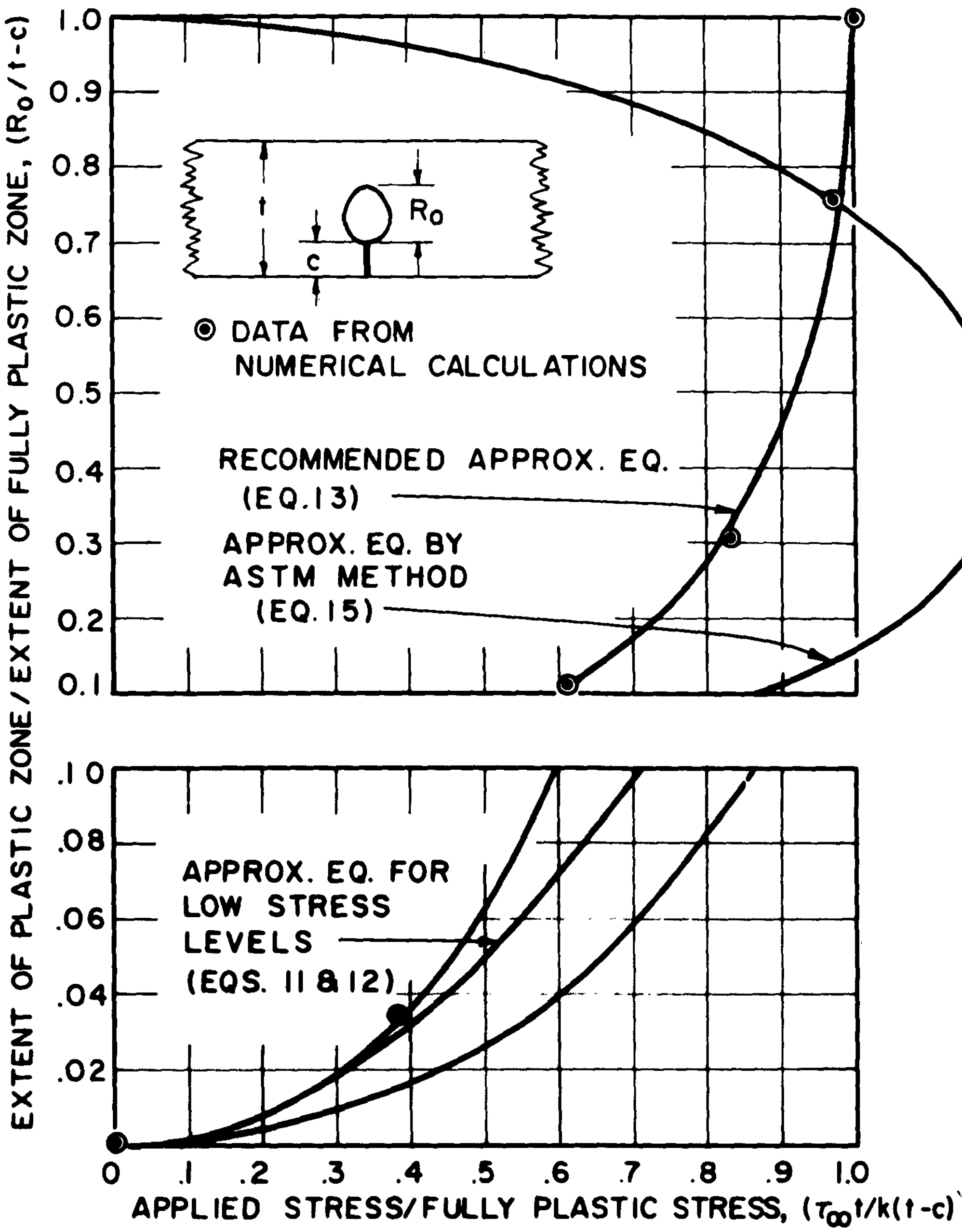

FIG.3 EXTENT OF PLASTIC ZONE VS. APPLIED LOAD, WITH APPROXIMATE EQUATIONS (c/t = 1/4).

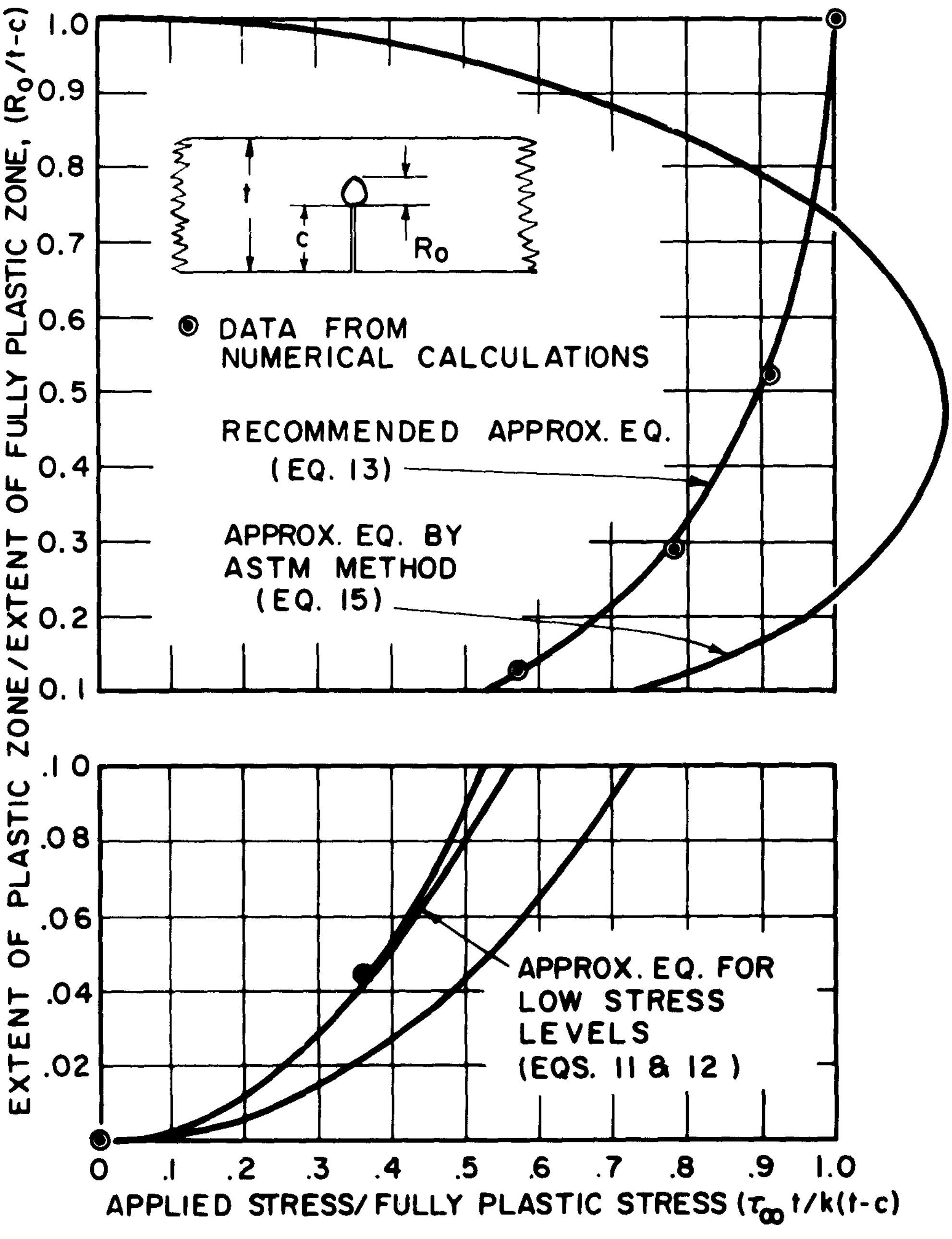

FIG. 4 EXTENT OF PLASTIC ZONE VS. APPLIED LOAD, WITH APPROXIMATE EQUATIONS (c/t = 1/2)

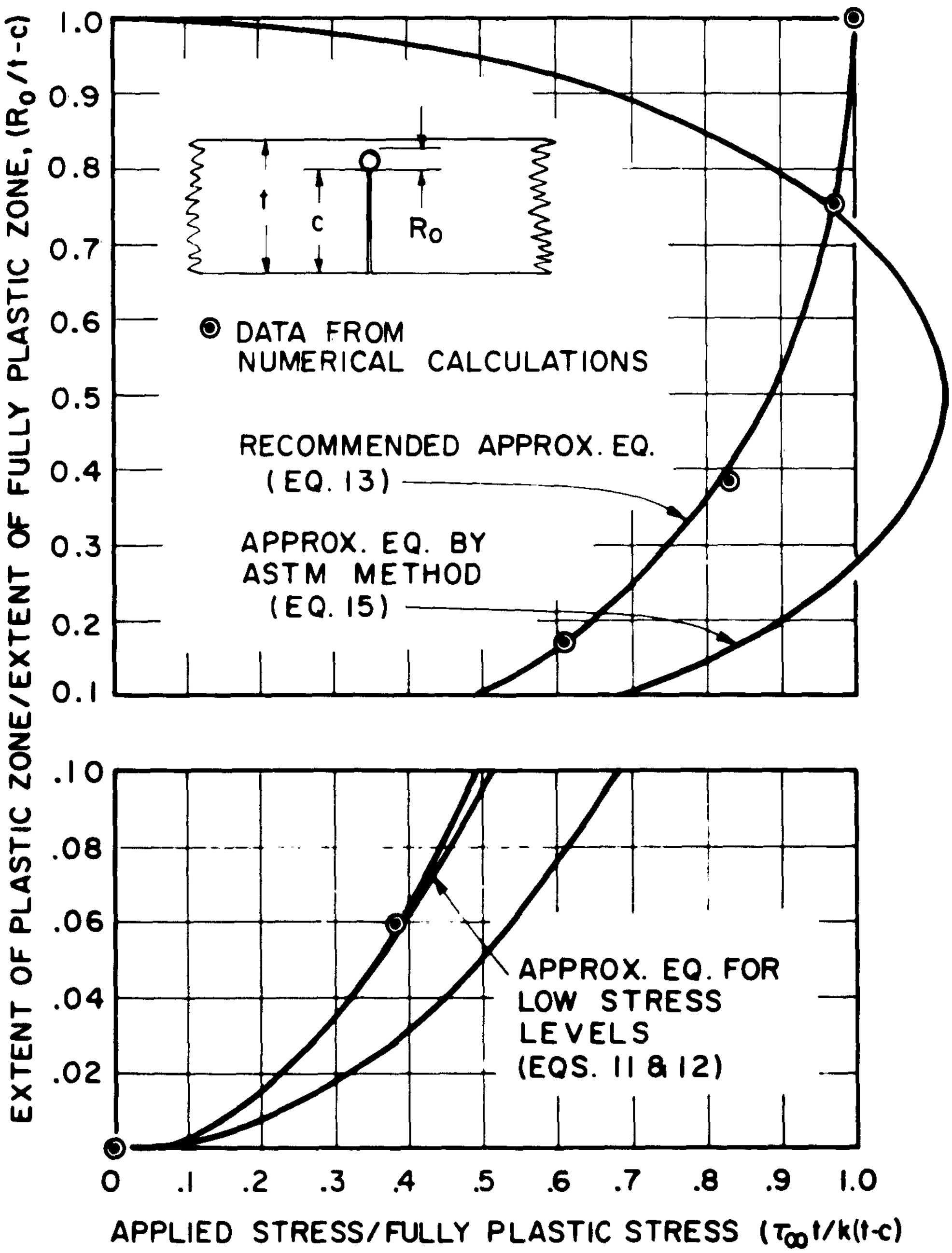

FIG. 5 EXTENT OF PLASTIC ZONE VS. APPLIED LOAD, WITH APPROXIMATE EQUATIONS (c/t = 3/4)

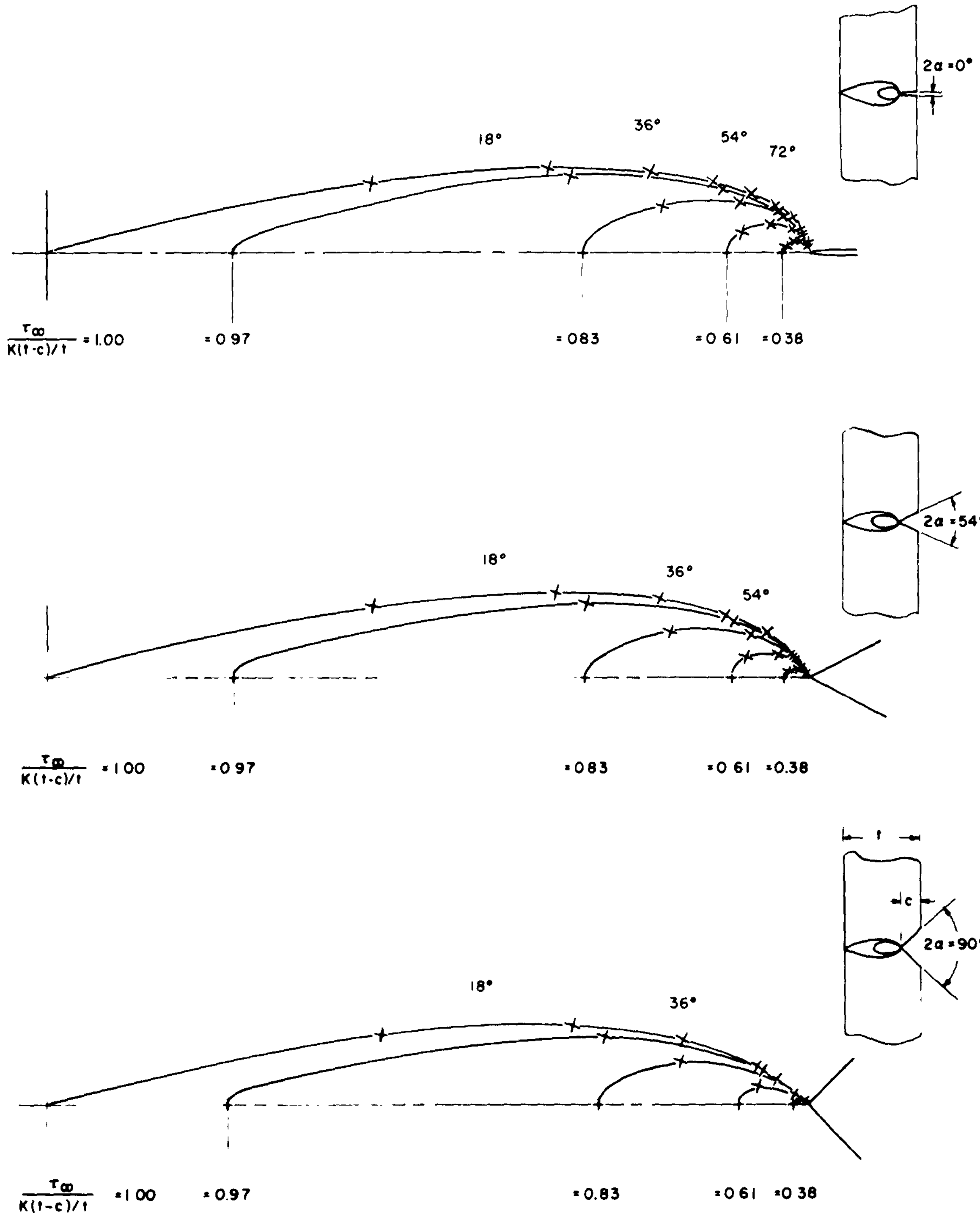

FIG. 6 ELASTIC-PLASTIC BOUNDARY AT VARIOUS APPLIED LOADS AND NOTCH ANGLES FOR $c/t = 1/4$.

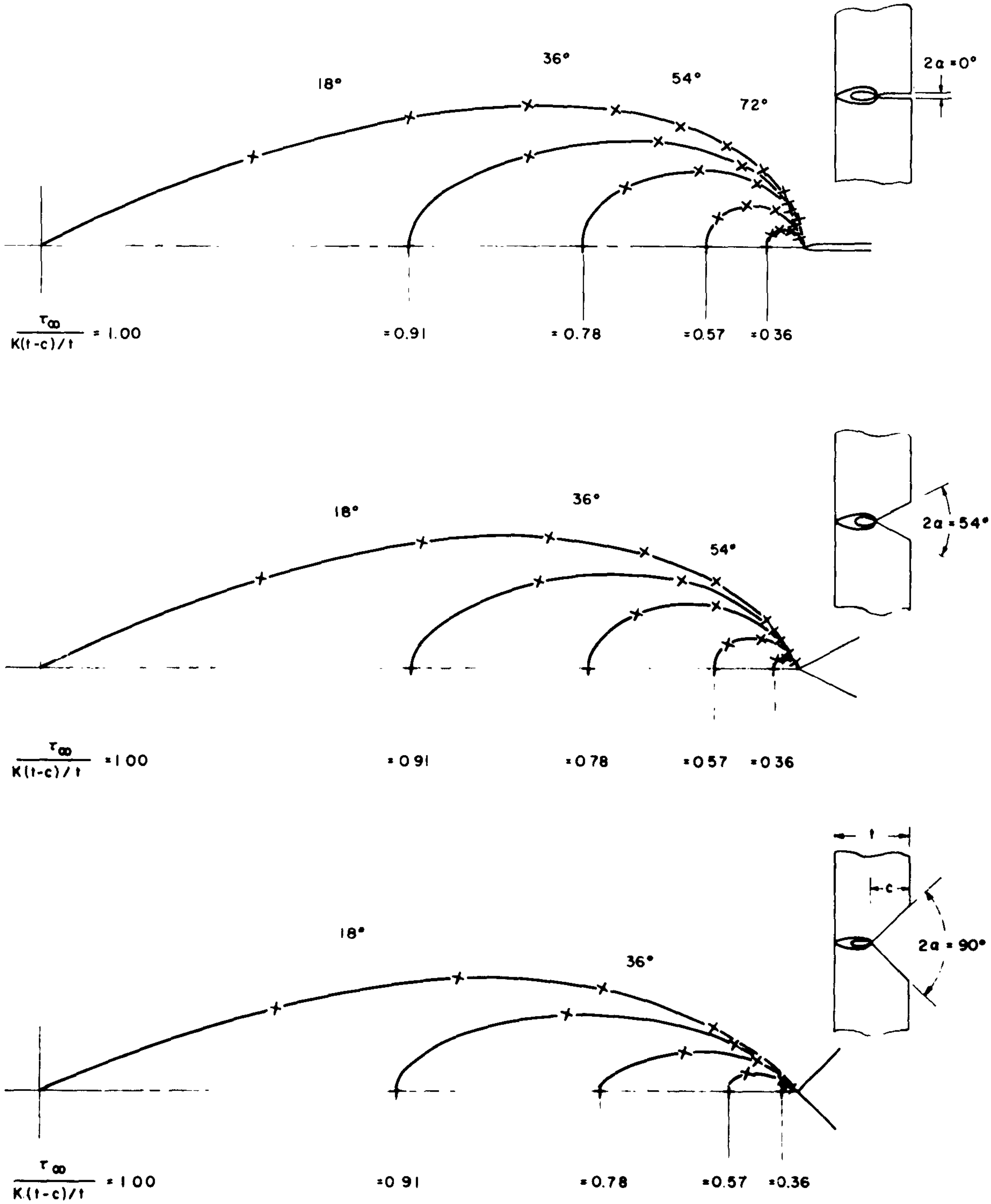

FIG 7 ELASTIC - PLASTIC BOUNDARY AT VARIOUS APPLIED LOADS AND NOTCH ANGLES FOR $c/t = 1/2$

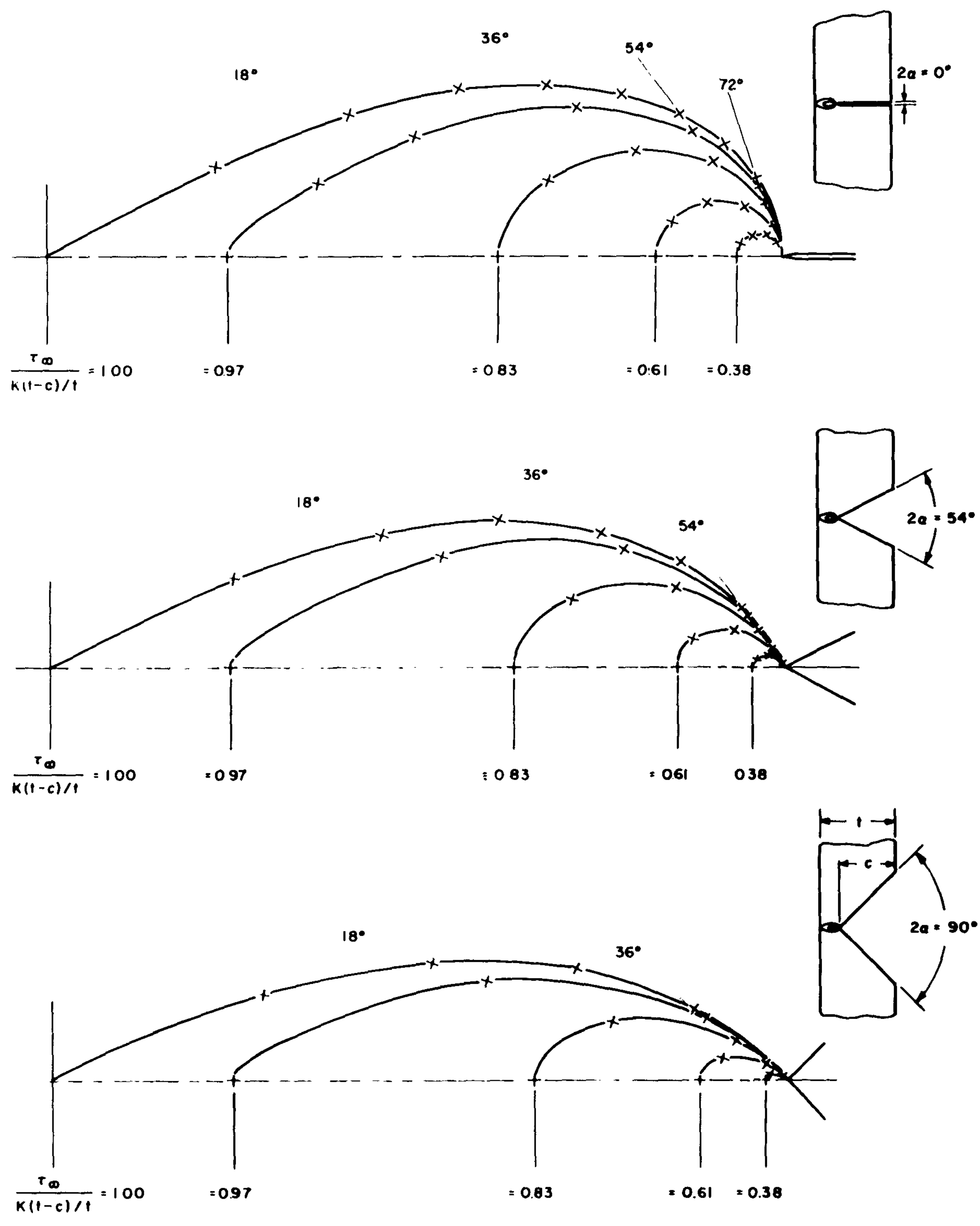

FIG 8 ELASTIC-PLASTIC BOUNDARY AT VARIOUS APPLIED LOADS AND NOTCH ANGLES FOR $c/t = 3/4$.

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Unclassified report

The development of the plastic strain in a V-grooved flat plate under longitudinal shear was followed from the elastic through the partially plastic to the fully plastic condition for a non-strainhardening material. The region of plastic flow develops monotonically. Adjacent to the zone of deformation in the fully plastic

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